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MEMORANDUM REPORT NO. 1421
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ON THE SCALING OF ROCKETS

Serge J. Zaroodny

Department of the Army Project No. 503-03-009

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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SJZaroodny/bj
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ABSTRACT

The theory of scaling of rockets from one caliber to another is reviewed. The estimate of accuracy is divided into an estimate of sensitivities and an estimate of causes of dispersion. It is shown that - with some attention paid to the units used - the sensitivities may be unchanged by a change in caliber; but that the relative precision should be expected to be poorer with smaller rockets. A rule of thumb is proposed: absolute precision varies as the 0.6 power of caliber.

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I. FOREWORD

The preparation of this paper has been commenced in connection with the discussion of a proposal of miniature rockets; but the thoughts expressed here go much further back, and can be found in many textbooks and BRL reports, in which they are unfortunately scattered and in effect masked by the large volume of other material. It can hardly be denied that concurrently with the general growth of the literature on rocketry there is going on a process of forgetting or neglecting the fundamentals of the theory; so that in each particular job much effort has to be made to cull out of the general theory the few relevant essentials. The object of this paper, accordingly, is to review certain specialized aspects of the elementary theory of rockets.

II. INTRODUCTION

The principles of dimensional analysis and similitude play a very essential role in ballistics, but unfortunately are appreciated less widely than they ought to be. To show that a neglect of these principles does exist, it may be sufficient to allude to the frequent illiterate use of such expressions as "dimensionless number" and "dimensionless length". The fact is, of course, that any number must be dimensionless; else, it is not a mathematical number, but a physical quantity. A physical quantity, however, remains a physical quantity no matter in which units it is expressed. Thus, the length of a table may be 2 meters, 2000 millimeters, 3/2 of the table width, 1 of itself, $1/\pi$ of the circumference of a circle having this length as the diameter, some clumsy number of inter-molecular distances, or some clumsy fraction of inter-stellar distances; but it would still remain the length of a table. Thus what is often neglected is the very marriage of the mathematical number with the meaning of the unit of this number. From the mathematical point of view we are led to the Fuchsian analysis and the measure theory - which is entirely more than is necessary. From the administrative (which is really the inter-personal) point of view we are prone to disdain the dimensional analysis as a lowly matter of units to be used - indeed, the discussion of units is the first thing to be knocked out whenever we try to condense. The crux of the matter lies perhaps in the personal, psychological, point of view. We sense the reciprocal relation between the number and the unit, recollect with a guilty conscience that on some occasions the 2-meter length had turned up as .002 millimeters, and thereafter shy from considering things which are thankless because they are both disdained and can become very fancy mathematics. The neglected principles, however, return with vengeance - as popular misconceptions, and as an inadequate traditional theory. We get lost on the shortcuts, and must face the fact that the shortest way to our goal is through the use of a little patience.

The problem of the dispersion of rockets fire divides in two phases; the study of the imperfections by themselves, and the study of the sensitivity of the rocket's performance to these imperfections. The study of the sensitivity is a domain of the theory, and we shall show that the situation there is rather straightforward, though there are some deficiencies in our traditional rocket theory. The study of the imperfections by themselves is a matter of experiment, empirical inference and conjecture, and hence invites much misunderstanding; in this connection we shall propose a rule-of-thumb. The business of combining these two phases has some controversial aspects, too; but it is outside the scope of this paper.

III. THEORY OF DISPERSION

Thus we may state, with an apology to the reader, the well-known fact that in the customary linearized theory of the dispersion of rocket fire each deviation, say θ, is supposed to be a sum of the products,

$$\theta = s_1 c_1 + s_2 c_2 + \dots$$

wherein one set of factors, say c_1, c_2, ..., is a set of the <u>causes</u> of dispersion, (that is, the unavoidable minor imperfections in the manufacture and the launching of the rocket), while the other, s_1, s_2, ..., is a set of the <u>sensitivities</u> (also spoken of as the differential coefficients, etc.), that is, of the deviations per unit cause. For convenience of speaking the numbers s_1, s_2 ..., c_1, c_2 ... are usually viewed as the components of vectors $\underline{s}$ and $\underline{c}$, and the deviation θ is viewed as an inner (scalar) product $\underline{s} . \underline{c}$. Since there may be many such components (sometimes as many as 18, see reference 1), the "space" of these vectors generally cannot be visualized; but this "space" is merely a way of speaking, and usually only few of these components matter.

The components of $\underline{c}$ are diverse physical quantities (angles, distances, velocities, etc.); accordingly, the components of $\underline{s}$ have different (reciprocal) dimensions, so that the products $s_1 c_1 = \delta_1$, say, all have the dimension of θ. Furthermore, the units of c_1, c_2 ... are basically arbitrary; thus, the linear malalignment of the jet thrust may be expressed equally well in thousandths of a foot, in inches, or in feet; this would change nothing but the <u>appearance</u> of the columns of the numbers which represent the vectors $\underline{c}$ and $\underline{s}$. For this reason it is usually awkward to speak of the magnitude, and of the direction, of the vectors $\underline{s}$ and $\underline{c}$. Nevertheless it is generally well understood that a "large" vector $\underline{c}$ represents a rocket made and launched sloppily (more exactly, non-uniformly), and a "large" vector $\underline{s}$ represents a design which is unfortunately very sensitive to all sorts of unavoidable imperfections. Correspondingly, the ideal $\underline{s} = 0$ represents a hypothetical design which is completely insensitive to all imperfections (so that the rocket is accurate even though sloppily made and launched); while the ideal $\underline{c} = 0$ (which has meaning only when the rocket is a member of a group) represents a perfectly-manufactured, and perfectly-launched, rocket.

The components of $\underline{s}$ can be computed; in fact, the set of these numbers is the principal result of the theory. A theory of the lateral dispersion of rocket fire amounts to a system of differential equations the trivial solution of which (the situation where all dependent variables are identically zero) represents the ideal flight of the rocket. The non-trivial solutions are "set off" by various disturbances, which are the "causes". These may be imperfections in the manufacture of the rocket - so to say, internal distortions (in which case they are called malalignments, and appear as the inhomogeneities of the equations); or any imperfections in the launching - so to say, external disturbances (in which case they are called mallaunchings, and appear as the non-zero initial conditions of the homogeneous equations). The linearity of the system is naturally required in order that deviation due to the various disturbances could be simply superposed.

In principle the components of $\underline{c}$ can be measured; but such measurements are difficult, and in practice, impossible. In fact, the vector $\underline{c}$ has a meaning only statistically. In this connection it is customary to use many mathematical shortcuts of speaking, which unfortunately often result only in a further estrangement of the theory and practice of rocketry. At this point it seems opportune to take the bull by the horns and restate these shortcuts of speech; for the experience shows that attempts at brevity all to often lead only to misunderstanding and discouragement.

We fire a group of rockets, determine the center of impact, and define θ as the deviation from the mean of the group. Thus we are thinking of the structure of the dispersion of the group as a matrix

$$\theta_1 = s_1 c_{11} + s_2 c_{21} + \dots + s_k c_{k1} + \dots$$

$$\theta_2 = s_1 c_{12} + s_2 c_{22} + \dots + s_k c_{k2} + \dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\theta_j = s_1 c_{1j} + s_2 c_{2j} + \dots + s_k c_{kj} + \dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Thus the column of θ averages to zero by definition. Thereby we are really defining each individual c_{kj} not as the actual cause of dispersion as we had visualized it at first, but as the deviation of that cause from the mean of the group. Thus each column of c (e.g., c_{11}, c_{12}, ...) averages to zero by definition.

There are now entirely too many c's for comfort, and we need shortcuts. It is natural to think of each column of c as another vector (e.g., $c_{\underline{1}}$, $c_{\underline{2}}$ ...). The "space" of such vectors is not the same as the space mentioned before: the number of its dimensions is the number of rounds in the group, rather than the number of causes of dispersion which we are considering. This statistical space, in fact, is much neater mathematically. There is no longer any question that the individual components of each vector have the same dimension. The magnitude of each vector (the square root of the sum of the squares of its components) is clearly proportional to the standard deviation of this variate. It is particularly interesting to note that if we consider any two vectors in this space, the cosine of the angle between them is, by definition, the correlation coefficient between the corresponding columns of numbers. It may also be noted that the vectors are confined to certain "multidimensional planes" in this space*.

Now enter Statistics; but with it there almost invariably sneaks in the assumption of the statistical independence of the causes of dispersion.

We compute the sum of squares of θ, and determine the standard deviation of θ, say σ_θ. In principle, there exist also the standard deviation of c_1, c_2, etc. The question then arises: even though we know practically nothing about the individual c_{kj}'s, could we not determine at least σ_{c_1}, σ_{c_2}, etc.? For this purpose one almost invariably writes the variance σ_θ^2 as the sum of squares:

$$\sigma_\theta^2 = s_1^2\ \sigma_{c_1}^2 + s_2^2\ \sigma_{c_2}^2 + \ldots$$

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* In statistical parlance, the imposition of the requirement that the mean of a column is zero reduces the "degree of freedom" of the group by one; or, the number of dimensions can thereby be reduced by one. This is easy to see in the case when we fire only three rockets, so that the space with the rectangular coordinates θ_1, θ_2, θ_3 can be easily visualized. The point in this space which represents these three numbers is confined to the plane $\theta_1+\theta_2+\theta_3 = 0$, hence can theoretically be specified by two numbers. In fact, two of θ_1, θ_2, θ_3 can be specified arbitrarily; the third is thereby fixed. It is usually easier, though, to think of the space in which the number of dimensions is the number of rounds in the group, rather than the number of the degrees of freedom.

Since the deviation due to an individual cause, $\delta_k = s_k c_k$, obviously has the standard deviation $\sigma_{\delta_k} = s_k \sigma_{c_k}$, the above sum of squares is easily recognized as

$$\sigma_\theta^2 = \sigma_{\delta_1}^2 + \sigma_{\delta_2}^2 + \ldots$$

The first-mentioned vector space (the space of the causes of dispersion) thereby acquires new, statistical, significance: σ_θ is simply the magnitude of the vector in the σ_δ-space. If it is at all possible to evaluate the variability of the deviation θ due to an individual cause of dispersion, the knowledge of s_k gives us an idea about σ_{c_k}.

This completes, essentially, the outline of the customary theory of dispersion. A strong exception could be taken, however, to the assumption of the statistical independence of the diverse causes. If we do not accept it, the last-written formula should be augmented on the right-hand side by a stack of terms of the type

$$+ 2\, \sigma_{\delta_1} \sigma_{\delta_2} r_{12}$$

where r_{12} is the coefficient of correlation between the (unknown) columns of c_1 and c_2. There is a considerable amount of ballistic experimental evidence that many such correlations are quite significant, in which cases the mechanical application of the assumption of the statistical independence may give a highly misleading understanding of the mechanism of dispersion. The subject is extremely interesting and promising, but complicated. For our present purposes we needed only the sharp division of the problem of the dispersion in two phases: the study of the senitivities and the study of the causes of dispersion.

IV. REVIEW OF THE THEORY OF SCALING

The general problem of scaling is discussed in a number of texts, such as reference 2, wherein the possible models of an experiment are classified as Adequate, Distorted and Dissimilar (viz., mathematical). Incidentally, the impossibility of the perfect scaling of an experiment has been expressed by Lord Kelvin as "the proper scale for an experiment is twelve inches to a foot".

The scaling of a free flight of projectile has been discussed from this point of view in reference 3. The scaling discussed there is principally such as arises out of a desire to facilitate the observation of flight (as thus is basically a study of the possible extensions of the famous Kent's pop-gun experiment). The basic problem is that of the existence of gravity, because of the constancy of which the simulation of the Mach number throughout the trajectory turns out to be not possible. An adequate model of a relatively small range of applicability had been found, however, in the scaling which preserves the Froude number u^2/gd, where g is the non-scalable acceleration of the gravity (u is a velocity such as the muzzle velocity, and d is a distance such as the caliber). In such scaling velocity is changed in proportion to the square root of the caliber.

Situation, however, is quite a bit different with the "lateral" dispersion of <u>rockets</u>. The processes responsible for this dispersion do not involve the gravity, and it is therefore possible to preserve such Froude number in which the acceleration of gravity is replaced by the acceleration of the rocket (due to thrust or drag). Since such acceleration is inversely proportional to the caliber - this is a point to which we shall presently return - the velocity in such scaling remains unchanged, and hence the variation of Mach number is properly simulated. The imperfection of such scaling lies in the fact that the Reynolds number is not simulated, and hence the phenomena related to the viscosity of the air, such as the Magnus torque, as well as the effects of gravity, cannot be simulated properly. Nevertheless, this appears to be an adequate model. A concise, and somewhat novel, theory of the phenomenon is given in reference 4; for reader's convenience it is further condensed in the Appendix to this paper; and other related discussions are given in references 5 and 6. The theory of s 'n-stabilized rockets is radically more complicated than that of the non-spinning finned rockets, and in spite of the extensive work by Hasenfus, Cell, Sacco et al - the relevant references are too numerous to be quoted here - no equivalent simplification of the theory seems to exist. Yet in principle the situation is not too different, and such an extension can be made. For our present purposes a complete extension of this theory is not necessary, and our conclusions can be viewed merely as just another exercise on the theory of reference 4; in fact, the full theory of that report is not necessary, either.

In brief, in our problem we need to assume only that unchanged are: (1) the geometric proportions and (2) the properties of materials. The conclusion of the theory then is that the whole mathematics of the lateral dispersion of rockets, all numbers associated with the (standardized) solution of the differential equations, and hence ALL SENSITIVITIES, will be unchanged if the units of distance AND of TIME are changed in proportion to the caliber.

We shall now prove and discuss these relations. The following table shows to what power of caliber are the various physical quantities changed as we pass from one size of the rocket to another*.

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* Here the physical quantities are understood to be expressed, of course, in the same absolute units, such as pounds, feet and seconds. The point of a proper scaling is that one uses specialized units, different for the two rockets, and such that all of these quantities are given by the same numbers for both rockets.

Quantity	Power of caliber
Proportions and angles	0
Properties of materials	0
Distances	1
Areas	2
Volumes, masses and weights	3
Pressures and stresses	0
Forces (other than gravity)	2
Accelerations (other than gravity)	-1
Times	1
Velocities	0
Moments (of force)	3
Moments of inertia	5
Angular accelerations	-2
Angular velocities	-1
Momenta (linear)	3
Angular momenta	4
Energies	3
Mach number (here considered essential)	0
Reynolds number (reckoned unimportant here)	1

Let us now run through this table briefly.

The first two lines are merely the statement of the assumptions. That the distances in the rocket are proportional to the caliber is obvious; but that the same applies to the length of launcher, travel to burnout and wave length of yaw will become clear presently. Next two lines are obvious. As to the pressure, we need only postulate that it depends upon the ratio of the burning surface to the throat area. Hence the thrust is proportional to the square of the caliber (and presently, when the invariability of the velocity is proven, the same will obviously hold for the drag and lift). Now, we have the acceleration inversely proportional to the caliber; but since the burning rate is unchanged, and the web thickness is proportional to the caliber, the time of burning is proportional to the caliber, too. From this the invariability of velocity follows. The remainder to the table is rather obvious; but we should note particularly that the angular velocities turn out to be inversely proportional to the caliber. This will trouble us somewhat, since the "mallaunching", or the angular velocity of the yaw, is traditionally considered as independent cause of dispersion. We need only note now that the travel to burnout is proportional to caliber, and that the same holds for the wave length of yaw (since the frequency of the yawing motion is proportional to the square root of the angular acceleration per unit angle), as well as for the travel on the launcher.

With a spin-stabilized rocket we may consider the principal causes of dispersion are the following five:

linear malalignment of the jet thrust	(distance or moment)
dynamic unbalance	(crossproduct or inertia)
static unbalance	(distance or moment of mass)
crosswind	(velocity)
mallaunching	(angular velocity)

Of these, the first three can easily be reduced to angles. Thus, we replace the linear malalignment by the "inclined nozzle" (cf. reference 5); the dynamic unbalance, by the angle between the geometric axis of the rocket and the principal axis of inertia; and the static unbalance, by an obvious ratio, which is equivalent to an angle. Let us view the response to these five types of disturbances as a deflection, that is, an angle. We notice at once that the response of both a big and a little rocket to a unit amount of the disturbance of these first three types is precisely the same.

Next, we notice that the response to the unit crosswind - which is a velocity and hence is invariant in this scaling - is unchanged, too.

Finally, we run into a little trouble with the mallaunching. Suppose we have a complete theory for the 4.5" rocket. This means we have the deflection, say θ mils, per unit of mallaunching. What is this unit? Let the units be meter for distance, second for time, kilogram for mass, newton for force, and mil for an angle. The unit of mallaunching then is the angular velocity of 1 mil/second, and the corresponding sensitivity properly is θ mil/(mil/second), which can be spoken of as θ seconds. Let us apply this theory to a homologous rocket of the caliber .45 inch. Our theory states that we shall have precisely the same sensitivity, θ mils per unit of mallaunching; but this unit is now different. The new units are, respectively (see the table): decimeter, decisecond, gram, centinewton and mil. Note that the velocities, densities and pressures are unchanged numerically: 1 dm/dsec = 1 m/sec, 1 kg/m^3 = 1 g/dm^3, 1 n/m^2 = 1 cn/dm^2. The new unit for angular velocity is 1 mil/decisecond, which is 10 mil/second. Hence any fixed angular velocity (say that due to the rotation of the earth) is now given by a number one tenth as great as before; and the deflection of the rocket due to that angular velocity is one tenth of what it was for the big rocket. Equivalently, if we prefer (as we might naturally do, although it would be slightly less neat mathematically) to use the conventional unit, the second, with the little rocket, too, we could say that the new sensitivity is θ deciseconds, or 0.1 θ seconds. Thus the scaling to a smaller caliber may seem to wipe out a bothersome cause of dispersion.

Unfortunately, this advantage is artificial and illusory. To show this, it might be simplest to return to the linear malalignment and dynamic unbalance. When we agreed to express these two causes of dispersion as angles, the theory stated that the deviations of both the big and the little rocket per unit of each of these two angles are the same. If we did not so agree, we would have had the sensitivity of the big rocket as, say, λ mils per 1 cm of linear malalignment, and say, μ mils per 1 kg-m^2 of the crossproduct of inertia of the dynamic unbalance. For the little rocket the sensitivities would then be λ mil/mm and μ mil/(gram-dm^2), that is, 10 λ mil/cm and 100,000 μ mil/(kg-m^2); but it would be foolish to say that the little rocket is more sensitive to the linear malalignment, and alarmingly more sensitive to the dynamic unbalance! We may be supposed, after Voltaire, to defend the right of the people to use whatever units they like, even though we deem these units unwise; but the fact remains that some units are convenient, and some are awkward; and the units of 1 kg-m^2 for the dynamic unbalance, and of 1 mil/sec for mallaunching here obviously are awkward units. With the linear malalignment and the dynamic unbalance we got rid of the awkwardness only because we understand and visualize these imperfections well enough so that we can recognize the more relevant unit for them, the mil; the trouble with the mallaunching is that we do not really understand the mechanism by virtue of which the mallaunching may arise. In fact, the appearance, among the presumably-"primary" causes of dispersion, of a quantity which is not invariant to this scaling attests simply to the primitive state of our theory. The fact that the other variables scale properly is nothing to brag about. The object of a theory is precisely to find that which is common to all (or in our case, both) rockets, and the constancy of deviation per unit of cause is to be expected.

V. SCALING OF THE IMPERFECTIONS IN MANUFACTURE

We have so far defended the scaling of the sensitivities of the rocket, having qualified this defense by pointing out that the causes of dispersion must be expressed in appropriate units. Let us now consider whether the imperfections, expressed in these units, may be the same for both little and big rockets.

There are those who apparently believe that the relative precision of the manufacture may be the same; that is, that if the standard deviation of the inclined nozzle for a big rocket is, say 1 mil, the same standard deviation can be expected from a little rocket. The reason for such a belief is obviously that the tolerances on a small gadget may be more exacting than on a big one. However, one can hardly believe that tolerances can be reduced as fast as in proportion to the caliber. It is an interesting question, then, with what power of a dimension can the tolerances on that dimension change? Is there a rule of thumb - or can one be made up?

Properly, such questions ought to be asked of someone who is in the thick of modern production methods (Picatinny? Frankford? ASME? Am. Soc. of Tool & Mfg. Engrs? Engineering Schools?). For the purposes of this paper, however, we propose that an "absolute" tolerance may run as the .6 power of the dimension; or equivalently, that the "relative" precision - viz., the ratio (tolerance/dimension) would vary inversely as the .4 power of the dimension. Then a 10-fold change of scale would change the tolerances by a factor of $10^{.6} = 4$; 20-fold, by a factor of $20^{.6} = 6$. Equivalently, the relative precision will be degraded, respectively, by the factors $10/4 = (10^{.4}) = 2.5$ and $20/6 = (20^{.4}) = 3.3$*.

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* These figures rather lean in favor of miniaturization. It would be simpler, but more pessimistic, to assume simply a square-root law. Then for a 10-fold scale the tolerances would change by a factor of 3.2, for the 20-fold, by the factor of 4.5. The degrading of the relative precision will be given by the same numbers.

The remainder of this section is merely a derivation and the defense of this rule of thumb. It is limited to the first three causes (linear malalignment, dynamic unbalance, and static unbalance), which are indeed in the nature of manufacturing imperfections, and which can be expressed as angles. These three causes are probably correlated, and it might not be too far from truth to consider them as one, and rather the principal, cause - at least for our present purposes. Then we could say, roughly, that the 4.5" rocket having a dispersion of 10 mils, if scaled precisely to the caliber .22" will have a dispersion of 33 mils*. The fact that it cannot be scaled exactly, will be considered separately.

On the attached sketch the abscissa indicates the size of a work, that is, a linear dimension; the ordinate is a "tolerance". The simplest "law" might be the line of the precision of 0.1%, passing through a typical standard of good machine-shop work: a tolerance of .001" on an 1" size. By such a law a building 100 ft long could be laid out only to a precision of 1 inch; but every carpenter and mason can do better than that. Similarly, by such law one might expect that a wire of .010" diameter could be made to a tolerance of .000,01"; this would be entirely unrealistic. A relative dimension is essentially an angle, and it is a basic fact that it is more difficult to measure an angle with a smaller object. One can then simply mark on such a chart a series of points representing one's idea of the precision possible with objects of various size, and fit a line with some reasonable round value of the exponent.

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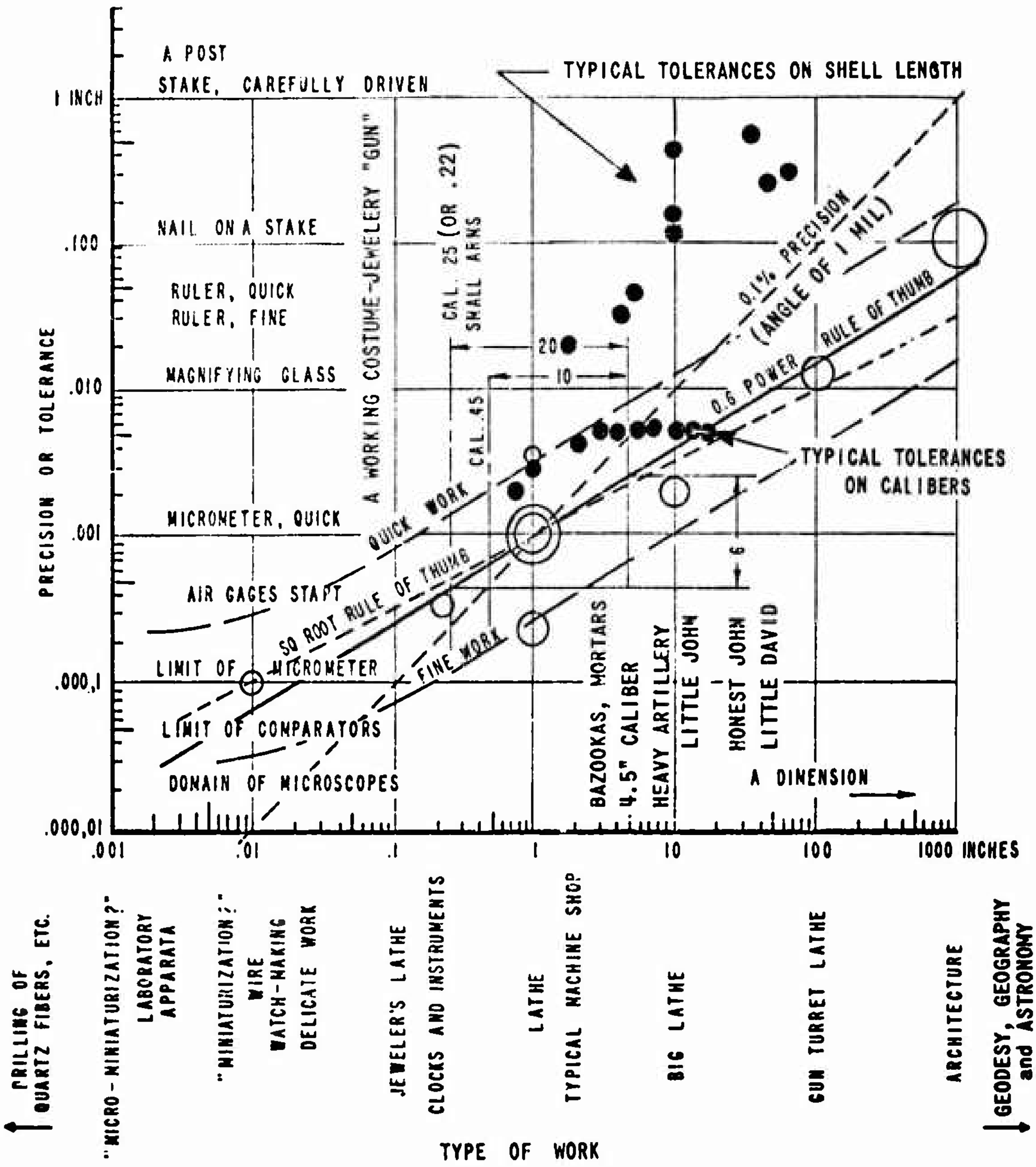
* We are assuming here that these three angles will change, from one caliber to another, as the relative errors of the linear dimensions. These three angles are not actually measured, but are only expressions of a certain combination of the errors in the linear dimensions.



A CRUDE SUMMARY OF GENERAL EXPERIENCE ON THE RELATION BETWEEN THE PRECISION AND THE SIZE OF WORK. THE SUGGESTED CONCLUSION IS THAT FOR AN EQUAL "QUALITY" OF WORK (AS HEREBY DEFINED), THE PRACTICAL TOLERANCES MAY BE EXPECTED TO VARY AS 0.6 POWER OF THE SIZE.

There is an unavoidable vertical spread, representing the quality, rather than the "type" of work (as defined on the abscissa), and representing also the general progress; but this spread seems to be surprisingly small in comparison with the general range of the variation of the variables. The most important concept - our judgement of the confidence which should be placed in the quality of the work of a particular manufacturer - is well within this spread. It is precisely in order to evaluate the probable effect of scaling as such, independently of the competitive claims of the manufacturer, in order to formulate more exactly the qualifying phrases "other things being equal", or "to the degree of workmanship well known to those who are versed in the art", or "with the same care with which other ordnance materiel is customarily produced", etc., that our rule of thumb is made.

The writer imagines that this type of generalization is of a sufficient basic importance that it has already been made by many others, and more competently. He particularly fears his lack of competence in the methods of miniaturization, on which remarkable progress has been made in recent years.

VI. SCALING OF CROSSWIND

Very little is known about the non-uniformity of the crosswind, and about the precision with which it can be measured. In fact, some of the existing knowledge on this subject is controversial. In such circumstances it may be natural to assume that the errors in the estimate of crosswind for the little rocket are precisely the same as for the big one. However, this would amount to also assuming wind to be uniform throughout the burning distance; but this is not very credible, either.

The range of our uncertainty includes the degradation factor 3.3 which we might assign to the manufacturing imperfections; hence it would not be straining our credulity too much to assume such degradation for the crosswind, too. In fact, some argument may be made up for this degradation of the uniformity of the wind. With a large rocket the wind is measured, and we are concerned only with errors of this measurement; with a small rocket there will probably be no measurement, and the variate of interest is the wind itself. We know in general that the wind profile exhibits turbulence of all scales; but the gustiness of the surface wind is pretty well recognized, while the evidence on the gustiness aloft (such as the presence of vortex rings in the updraft "thermals") is scant. A large rocket is more likely to be fired at a fairly high elevation, into the regions of large-scale turbulence; a small rocket is likely to be fired close to the ground, between the trees, etc. If a small rocket is in effect a fragment of a large and long-range projectile, there would arise another important source of dispersion that is ordinarily neglected with launcher-fired rockets: this is the initial yaw. Mathematically, the effect of the initial yaw is very similar to the effect of crosswind, and one might make a crude allowance for it simply by degrading the uniformity of the crosswind.

It therefore seems reasonable to assume a degradation of the uniformity of the wind to the same extent to which we assumed the degradation of the manufacturing precision.

VII. SCALING OF THE MALLAUNCHING

In section IV we have noted that if we agree to express the mallaunching in the appropriate units, the sensitivity of the smaller rocket to the mallaunching will remain unchanged (and that the apparent reduction of this sensitivity was an illusion resulting from the use of the arbitrary and awkward units). We have also noted the absence of a real understanding of the mechanism through which this supposedly-primary cause of dispersion is incurred.

We may also note that the yawing frequency of the rocket changes inversely as the caliber. If we assume that the mallaunching (an angular velocity) is proportional to this frequency, the apparent reduction of the sensitivity is wiped out.

Even if we had struck out this cause entirely, not much improvement of the dispersion would result. This cause is only one out of five; and if we assume that all five contribute equally, the change in the standard deviation would be of the order of only 10% (since $.8^{1/2} = .9$); in practice, this could not be detected.

In fact, some argument can be made for a degradation of the mallaunching with decrease in caliber. If the mallaunching is, in some involved way, a result of the linear malalignment, static and dynamic unbalance, imperfection of the launcher rails, etc., it would obviously be degraded in the same way as those other causes. If the mallaunching is a result of forces imposed upon the rocket from outside (the trembling of a hand-held launcher, vibration of a riding vehicle, etc.), whose frequency does not depend upon the natural frequency of the rocket, and whose magnitude is independent of the size of the rocket, the angular velocity of a small rocket may go up as its acceleration, inversely to the square of the caliber. (See table in Section IV) If the little rocket is a result of "submissiling" of a large one, a new and drastically important increase in the mallaunching can be expected.

It seems quite conservative to assume the same degradation factor 3.3 for the mallaunching - or, equivalently, to strike out this cause from the consideration altogether.

VIII. EFFECT OF THE NON-HOMOLOGOUSNESS OF THE DESIGN

All of the above reasoning - which has given us the degradation factor of 3.3 - has been based on an impossible fiction: an exact scaling of the design of the rocket from 4.5" caliber to the caliber .25". Actually, of course, there will have to be some radical change in the proportions of the design, and in the materials used. At this point it is very tempting to insist that someone go to the trouble of computing the changes in the sensitivities resulting from the non-homologousness of the design: for the change in the suitably-scaled sensitivities is really the only information that the theory can furnish. This may be rather a red herring. There seems to be little wrong in assuming that after a great deal of fussing over the <u>design</u> (not over the tolerances!) of the little rocket it will be possible to achieve roughly the same sensitivities, expressed in the suitable generalized units, which are associated with the "nominal" 10-mil dispersion of a rocket in the 4.5" caliber. The additional degradation will still be a result of the tolerances; but it will be more that our allowed factor of 3.3 because the nature of the manufacture will be different.

The anticipatable examples of such additional degradation are many, and only very few of them need be given here.

The tolerances of setting a tungsten-carbide tool on an automatic lathe are replaced by the tolerances on the manufacture, adjustment, maintenance, wear and discarding of the drawing and crimping dies and mandrels.

Forging followed by machining is replaced by cold forming and swaging, and there is a strong temptation to replace a fairly hard steel with softer materials and even with plastics. The "creep" of all of these materials is generally much greater than with steel.

The screw-thread joint of the nozzle assembly would probably be replaced by a crimping or cementing of a small nozzle plug in a thin-walled tube.

A control of the progressiveness of the propellant configuration will be much more difficult. There will be a temptation to rely upon the erosion of the nozzle, so as to prevent the blow-up of the little rocket in the later stages of burning. There will even be, accordingly, a temptation to use a more-rapidly-eroding material for the nozzle. There seems to be hardly any question that with purposely-quickly-eroding nozzles there would arise a host of problems which were of small importance with large rockets.

Both the metals, and particularly the modern rocket propellants are essentially crystalline in their structure. Since all sort of imperfections may be traced, to some extent, to the imperfections of the crystalline structure of the rocket components, a very serious degradation of performance is to be expected. In fact, for such imperfections the 20-fold reduction of the scale should mean a 20-fold degrading of the relative precision.

A crude figure, a factor of 2, may be rather conservative. Thus a typical 10-mil dispersion of the 4.5-inch rocket, which would become 33 mil on the assumption of the hypothetical homologous design in caliber .22 size, might be some 70 mils in reality.

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REFERENCES

1. Zaroodny, Serge J. Equations of Motion of a Tractor Rocket - a Crude First Attempt, BRLTN 1449, (1962).

2. Murphy, Glenn Similitude in Engineering, Roland Press, (1950).

3. Zaroodny, Serge J. Trajectory Models in Mortar Fire, BRLM 833, (1954).

4. Zaroodny, Serge J., Lesser, T. Unified Theory of Lateral Deviation of Unguided Finned Rockets, BRL 1132, (1961).

5. Zaroodny, Serge J. A New Theory of Dispersion of Unguided Finned Rockets and Self-Alignment of Rockets, BRL 1072, (1959).

6. Zaroodny, Serge J., Mylin, D. and McIntosh, F. Spin of Honest John Type of Rocket-Experimental Data and Their Implication for the Design, BRL 1090, (1959).

7. A Review of a Textbook, Review 12780, Mathematical Reviews, 22, 12B, (December 1961).

APPENDIX

A RESUME' OF THE THEORY OF LATERAL DEVIATION OF FINNED ROCKETS

(condensed from reference 4)

Omit gravity, spin, and all aerodynamic forces except the righting moment. Let the moment coefficient K_M, the air density ρ, the mass of the rocket M, its transverse moment of inertia B, the thrust F, be all constant. We need consider here only the linear malalignment L, that is, a constant spurious torque LF. Let d = caliber, u = velocity, t = time. Since the acceleration is G = F/M, we have u = Gt. Consider a "line of aim", and let z = the deviation of the rocket's c.g. from that line, and ϕ = the angle the rocket's axis makes with that line. The deflection (the angle which trajectory makes with the line of aim) is $\theta = \dot{z}/u$, the dot meaning d/dt. The aerodynamic yaw is $\delta = \phi - \theta = \phi - \dot{z}/u$; when u is small, the torque due to δ is negligible, anyway. The differential equations of motion are:

$$B\ddot{\phi} = K_M \rho \; d^3(Gt)^2(\phi - \dot{z}/Gt) + LF$$

$$M\ddot{z} = F\phi$$

Let X be a unit of distance, and Y a unit of time. Let z and t in these units be Z and T, and let d/dT be denoted by a prime. Let $B = MK^2$. Then z = ZX, t = TY, d/dt = (d/dT)/Y, and equations become

$$MK^2\phi''/Y^2 = K_M \rho d^3 G^2 Y^2 T^2(\phi - XZ'/GY^2T) + LF$$

$$MXZ''/Y^2 = F\phi$$

or

$$\phi'' = (K_M \rho \; d^3 G^2 Y^4/MK^2)\left[\phi - (Z'/T)/(GY^2/X)\right] + LGY^2/K^2$$

$$Z'' = (GY^2/X)\phi$$

It is obviously desirable to choose the units so that the term GY^2/X, which is the acceleration of the rocket in these units, be 1. The term involving L then becomes LX/K^2, which suggests that the logical unit for the small distance L is the distance K^2/X, visualizable as the center of percussion of the rocket with respect to a transverse blow delivered at the distance X from the c.g. of the rocket. It is furthermore desirable that the group of factors $K_M \rho d^3/MK^2$ be $1/L^2$, for then (taking L in our peculiar units as unity), the equations become

$$\phi'' = T^2(\phi - Z'/T) + 1$$
$$Z'' = \phi,$$

so that numerically the system of equations is precisely the same for all rockets. As a matter of fact, the distance $X = (MK^2/K_M \rho d^3)^{1/2}$ is well known in ballistics: it is (wavelength of yaw)/2π, or the distance travelled by rocket during 1 radian (the fraction $1/2\pi$ of the period) of the yawing motion.

For the purposes of this paper it is now interesting to show that the mathematics of the motion is not affected by a change of caliber for homologous designs.

Since M is proportional to d^3, while the radius of gyration K is proportional to d, the reference distance X is proportional to the caliber, too. So is the reference distance K^2/X, and (for strictly homologous design) the L itself. The essential result of the numerical solution of the equations is the deflection $\Theta = \dot{z}/u$; as we have seen in the derivation, this is also Z'/T, and hence unaffected by the change of caliber. It only remains to show that T at burnout is unaffected by the change of d, and neither is T at launch, for strictly homologous launcher. This point has been reviewed in the text.

Inclusion of the effects of other malalignments, of mallaunching, of crosswind, of "slow" spin, and even of the spin of the spin-stabilized rockets (see reference 6) involves much labor and algebra, but leads to substantially the same conclusion. The effect of the linear malalignment is described here merely as an example, and will suffice.

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